

FINITE ELEMENT SIMULATION OF SURFACE TENSION EFFECTS IN A HEATED THIN FLUID LAYER

Xiaowen Wang^a and Graham F. Carey^b

^aObject Reservoir Inc.
4301 Westbank Drive, Bldg B, Suite 200
Austin, TX 78746
swangken@objectres.com

^bTexas Institute for Computational Engineering and Sciences
The University of Texas at Austin
201 E. 24th Street, ACE 6.412,
Austin, TX 78712
carey@cfdlab.ae.utexas.edu

The present study deals with modeling and numerical approximation of thermocapillary and surfactant effects on long wavelength behavior of thin liquid layers. The character and long wavelength stability of such surface-tension-driven Rayleigh Bénard Marangoni convection in heated fluid layers has been a topic of recent experimental and theoretical interest [1]. In the present work, we first apply a depth-averaged perturbation analysis via long wavelength asymptotic expansion of the incompressible Navier-Stokes equations and coupled heat equation for the case of a horizontal layer or a layer at slight inclination. This is described by a 4th order nonlinear elliptic PDE for the elevation of the free surface. Next, we recast this as a second order system and construct a mixed finite element scheme. This formulation introduces an auxiliary curvature variable but reduces the regularity requirement on the finite element basis functions from C^1 to C^0 . The algorithm then proceeds by implicit time integration for the layer elevation with an embedded substitution solve for the auxiliary curvature variable. This implies that some care must be exercised in the initial field projection to ensure that the elevation and the curvature are projected consistently, particularly when using high order time integration schemes [2]. Verification convergence plots are carried out for the formulation, method and algorithm. A stability analysis is developed. In the case of the slightly inclined layer the PDE has an extra convective term that we show may give rise to premature onset of instability. Numerical studies of the long wavelength behavior of the layer elevation are conducted for both the horizontal and slightly inclined layer. The problem class, formulation and approximation are then extended to include the effect of an insoluble surfactant monolayer. This involves an additional transport equation and a stability analysis of this more complex system is also conducted [3]. Supporting numerical studies have been carried out to explore the evolution of the surface in concert with the transport of surfactant.

References

- [1] S. J. VanHook, M. F. Schatz, J. B. Swift, W. D. McCormick and H. L. Swinney, *Long-wavelength surface-tension-driven Bénard convection: experiment and theory*, J.Fluid Mech. 1997, vol. 345, pg. 45-78.
- [2] X. Wang and G. F. Carey, *Finite element study of a heated thin fluid layer*, ICES report, 2003 (in preparation).
- [3] X. Wang and G. F. Carey, *Finite element study of a heated thin fluid layer including surfactant effect*, ICES report 2003.